APPENDIX B - The Collision Risk Model for the Vertical Dimension

Part 1: Collision risk model on the same ground track at adjacent flight levels

$$N_{az} = P_z(S_z)P_y(0)\frac{\lambda_x}{S_x}\left\{E_z(same)\left[\frac{|\overline{\Delta V}|}{2\lambda_x}+\frac{|\overline{\dot{y}}|}{2\lambda_y}+\frac{|\overline{\dot{z}}|}{2\lambda_z}\right]+E_z(opp)\left[\frac{|\overline{V}|}{\lambda_x}+\frac{|\overline{\dot{y}}|}{2\lambda_y}+\frac{|\overline{\dot{z}}|}{2\lambda_z}\right]\right\}$$

The individual parameters that make up the model statement and their definition are as follows:

CRM Parameter	Description
N_{az}	Number of fatal accidents per flight hour due to loss of vertical separation.
S_z	Vertical Separation minimum.
$P_z(S_z)$	Probability that two aircraft nominally separated by the vertical separation minimum S_z are in vertical overlap.
$P_y(0)$	Probability that two aircraft on the same track are in lateral overlap.
λ_x	Average aircraft length.
λ_y	Average aircraft wingspan.
λ_z	Average aircraft height with undercarriage retracted.
$\hat{S}_x$	Length of longitudinal window used to calculate occupancy.
$E_z(same)$	Same direction vertical occupancy.
$E_z(opp)$	Opposite direction vertical occupancy.
$\overline{\|\Delta V\|}$	Average relative along track speed between aircraft on same direction routes.
$\overline{\|V\|}$	Average aircraft ground speed.
$\|\overline{\dot{y}}\|$	Average relative cross track speed for an aircraft pair nominally on the same track.
$\|\overline{\dot{z}}\|$	Average relative vertical speed of an aircraft pair that have lost all vertical separation

Same and opposite direction passing frequencies, $N_x(same)$ and $N_x(opp)$, are related to the same and opposite direction vertical occupancies through the following relations:

$$N_x(same) = \frac{\lambda_x}{\hat{S}_x} E_z(same)\frac{\overline{|\Delta V|}}{2\lambda_x}$$

and

$$N_x(opp) = \frac{\lambda_x}{\hat{S}_x} E_z(opp)\frac{|\overline{V}|}{\lambda_x}$$

where the parameters are identical to those described in the previous table.

An equivalent opposite direction passing frequency, as used in the Global System Performance

Specification, can be derived from the same and opposite direction passing frequencies using the following relation:

$$N_x(equivalent) = N_x(opp) + N_x(same)\frac{c_1}{c_2}$$

where:

$$c_1 = \left[1 + \frac{\lambda_x}{\lambda_y}\frac{|\bar{\dot{y}}|}{|\overline{\Delta V}|} + \frac{\lambda_x}{\lambda_z}\frac{|\bar{\dot{z}}|}{|\overline{\Delta V}|}\right]$$

and

$$c_2 = \left[1 + \frac{\lambda_x}{\lambda_y}\frac{|\bar{\dot{y}}|}{2|\bar{V}|} + \frac{\lambda_x}{\lambda_z}\frac{|\bar{\dot{z}}|}{2|\bar{V}|}\right]$$

Part 2: Collision risk model for the vertical dimension on the same ground track at adjacent flight levels applied to aircraft descending through flight levels without clearance

Two models are used for the determination of collision risk due to levels crossed without clearance. The choice of models depends on the assumed climb/descent rate. Slowly descending aircraft are assumed to maintain the same attitude as in level flight. Rapidly descending aircraft are assumed to have attitude changes that affect the angle at which the transitioning aircraft cross each flight level and hence the possible size of the collision envelope. Model 1 is employed for climb/descent rates less than or equal to 4000ft/min (approximately 40 knots) while Model 2 is used for emergencies such as pressurization failures which can result in descent rates in the region of 4000ft/min to 6000ft/min (approximately 40 to 60 knots).

Model 1: Climb/Descent Rates ≤ 4000ft/ min(≅ 40 knots)

To estimate the risk associated with aircraft descending through a track it is assumed that the lateral path-keeping performance is no worse than that for an aircraft in level flight. For aircraft descending through flight levels at rates that are consistent with model 1, the collision risk model is:

$$\hat{N}_{az} = P_y(0)\frac{2N_{fl}(s)}{T}\frac{\lambda_x\lambda_z}{S_x|\bar{\dot{z}}_1|}\left\{E_z(same)\left[\frac{|\overline{\Delta V}|}{2\lambda_x} + \frac{|\bar{\dot{y}}|}{2\lambda_y} + \frac{|\bar{\dot{z}}_1|}{2\lambda_z}\right] + E_z(opp)\left[\frac{|\bar{V}|}{\lambda_x} + \frac{|\bar{\dot{y}}|}{2\lambda_y} + \frac{|\bar{\dot{z}}_1|}{2\lambda_z}\right]\right\}$$

The caret over the symbol N_{az} indicates additional risk, T is total system flight time, $N_{fl}(s)$ is the number of flight levels crossed without clearance during slow descents and $\dot{z}_1$ is relative vertical speed for aircraft pairs in model 1 during the crossing.

Model 2: Descent Rates between 4000ft/min and 6000ft/min (≅ 40 to 60 knots)

Model 1 takes no account of the angle at which the transitioning aircraft crosses a particular flight level and assumes the collision risk between two aircraft of length λ_x, wingspan λ_y and height λ_z is equivalent to the collision risk between a particle and a rectangular box of dimensions $2\lambda_x \times 2\lambda_y \times 2\lambda_z$. This assumption is valid for slowly descending or climbing aircraft, but not for aircraft in rapid descent, e.g., during pressurization failure. Model 2, therefore, considers the paths of a rapidly descending aircraft and an aircraft in level flight and represents a collision between them as the entry of the descending aircraft's center into a "lozenge" surrounding the aircraft in level flight. The resulting expression for $\hat{N}_{az}$ is:

$$\hat{N}_{az} = \left(\frac{\lambda_x^2 \left(1 + \frac{\pi}{4} \right) + 2\lambda_x \lambda_z}{2T \hat{S}_x |\bar{\dot{z}}_2|} \right) N_{fl}(r) P_y(0) \left\{ E_z(same) \left[\frac{|\bar{\dot{y}}|}{2\lambda_y} + \frac{\sqrt{(\Delta V)^2 + \dot{z}_2^{\,2}}}{\lambda_{xz}(same)} \right] + E_z(opp) \left[\frac{|\bar{\dot{y}}|}{2\lambda_y} + \frac{\sqrt{(2V)^2 + \dot{z}_2^{\,2}}}{\lambda_{xz}(opp)} \right] \right\}$$

The above expression contains two new parameters λ_{xz} *(same)* and λ_{xz} *(opp)*. λ_{xz} *(same)* is the average length of the path followed by the descending aircraft's center as it traverses the "lozenge", when the aircraft are headed in the same direction. λ_{xz} *(opp)* is the average path length when the aircraft are headed in opposite directions. The values of these parameters need to be based on Asia Pacific aircraft size. For example, using a maximum assumed absolute relative longitudinal speed of 50 knots for aircraft in the NAT, values of λ_{xz} *(same)* and λ_{xz} *(opp)* have been calculated as 0.36143 and 0.0612 respectively. $N_{fl}(r)$ is the number of flight levels crossed without clearance during rapid descents and the symbol $\dot{z}_2$ is the relative vertical speed for aircraft pairs in model 2 during the crossing.

Part 3: Collision risk model for the vertical dimension on the same ground track at adjacent flight levels applied to aircraft adhering to incorrect flight levels

The proportion of the total flying time spent at incorrect levels, *Q*, is determined by summing the individual times for each large height deviation occurring at an integer multiple, *n*, of a full separation minimum and dividing by, *T*, the total system flight time. *Q* may be interpreted as the probability that an aircraft is flying at an incorrect level. To estimate the probability of vertical overlap during these events, *Q* is multiplied by the probability, $P_z(0)$, that two aircraft nominally flying at the same level are in vertical overlap. Therefore, the vertical overlap probability arising from deviations that are integer multiples of the vertical separation minimum is given by:

$$\sum_n P_z(nS_z) = P_z(0)Q$$

Having determined $\sum_n P_z(nS_z)$, the collision risk is determined by using the Reich Collision Risk Model presented in part 1 of this appendix.

Part 4: Collision risk model for the vertical dimension for intersecting routes at adjacent flight levels

The mathematical form of the collision risk model for intersecting routes at adjacent flight levels would be extremely complex if aircraft were assumed to have a rectangular shape as in part 1 of this appendix. To reduce this complexity aircraft shapes are assumed to be right circular cylinders. If a given route is crossed by another, the given route's rate of accidents with aircraft on the crossing route, expressed in accidents per flight hour is:

$$\hat{N}_{az} = \frac{2 P_z(S_z)}{kF} \sum_{j=1}^{N} n_j P_o(s_j) \left[1 + \frac{|\bar{\dot{z}}|}{2\lambda_z} \overline{d_o(s_j)} \right]$$

The new parameters in the above model for intersection routes are as follows:

k = the number of hours during which the intersection's traffic is monitored;

F = the given route's traffic flow expressed in flight-hours per hour;

$P_o(t)$ = the probability that two aircraft experience a horizontal overlap, given that: (1) one of the aircraft is assigned to the given route and the other to the intersecting route; (2) their assigned flight levels differ by S_z; (3) they have t hours difference between their estimated times of arrival at the intersection;

$\overline{d_o(t)}$ = the average duration of a horizontal overlap, given that: (1) one of the aircraft is assigned to the given route and the other to the intersecting route; (2) their assigned flight levels differ by S_z; (3) they have t hours difference between their estimated times of arrival at the intersection; and (4) they experience a horizontal overlap; with each other;

N = an integer chosen to be large enough so that $P_o(t)$ changes by no more than a (small) chosen percentage over each of the intervals

$$\left[\frac{(j-1)t_M}{N}, \frac{jt_M}{N}\right), \quad \text{for } j = 1,2, \ldots, N;$$

$$\frac{(2j-1)t_M}{2N} = \quad \text{the midpoint of the interval } \left[\frac{(j-1)t_M}{N}, \frac{jt_M}{N}\right]$$

n_j = (for j = 1,2,…; N) the number of pairs of aircraft that arrive in the vicinity of the intersection, during k hours of monitoring, with one aircraft assigned to each of the intersecting routes, with the aircraft assigned to flight levels separated by S_z, and with t, the difference between their estimated times of arrival at the intersection, in the interval

$$\left[\frac{(j-1)t_M}{N}, \frac{jt_M}{N}\right]$$

Part 5: Collision risk model for the vertical dimension applied to formation flights

The collision risk model for aircraft in a formation that are paired with typical aircraft at adjacent altitudes is again a modified form of the collision risk model for the vertical dimension as presented in part 1 of this appendix. When aircraft within a formation are paired with typical aircraft at adjacent altitudes the parameter values $2\lambda_x, 2\lambda_y, 2\lambda_z, P_y(0)$ and $P_z(S_z)$ used in part 1 for typical Caribbean and South American aircraft pairs require modification due to the increased volume of airspace restricted to aircraft within the formation.

Let the shape of formation be represented by a box of length, width and height Γ_x, Γ_y, and Γ_z, respectively. The modified parameters are given in the second column of the following table:

CRM Parameters for Typical Aircraft Pairs	Modified CRM Parameters for Aircraft in Formation Flight paired with Typical Aircraft
$2\lambda_x$	$2\lambda_x + \Gamma_x$
$2\lambda_y$	$2\lambda_y + \Gamma_y$

CRM Parameters for Typical Aircraft Pairs	Modified CRM Parameters for Aircraft in Formation Flight paired with Typical Aircraft
$2\lambda_z$	$2\lambda_{z} + \Gamma_z$
$P_y(0) = \int_{-\lambda_y}^{\lambda_y} \int_{-\infty}^{\infty} h(y)h(y+w)dydw$	$P_y(0) = \int_{-(\lambda_y+\Gamma_y)}^{(\lambda_y+\Gamma_y)} \int_{-\infty}^{\infty} h(y)h(y+w)dydw$
$P_z(S_z) = \int_{S_z-\lambda_z}^{S_z+\lambda_z} \int_{-\infty}^{\infty} f(z)f(z+w)dzdw$	$P_z(S_z) = \int_{S_z-\lambda_z-\Gamma_z}^{S_z+\lambda_z+\Gamma_z} \int_{-\infty}^{\infty} f(z)g(z+w)dzdw$

Comparison of CRM Parameters for Typical Aircraft Pairs and
Aircraft in Formation Flight Paired with Typical Aircraft

In the above table *h(y)* is the density function for lateral error, *f(z)* is the TVE density function for approved aircraft and *g(z)* is the TVE density function for aircraft within the formation.

Part 6: Collision risk model for the vertical dimension applied to aircraft in vertical alignment for the entire crossing at adjacent flight levels

Assume there are n route categories in a route system and that the average flight time for each category is $T_1, T_2, ..., T_n$. Let the number of flights during which two aircraft are in continual longitudinal overlap be $k_1, k_2, ..., k_n$. Then the additional risk on the entire route system can be expressed by the following equation:

$$\hat{N}_{az} = \frac{2}{T} P_y(0) P_z(S_z) \left[\frac{|\bar{\dot{y}}|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right] \sum_{r=1}^{n} k_r T_r$$

Part 7: Summary

The risk estimate in the vertical dimension is estimated as the sum of the risks in each of the six parts of this appendix. It is compared to the regional Caribbean and South American Target Level of Safety (TLS) of 5 fatal accidents in 10^9 flying hours which embodies the risk due to the loss of vertical separation from <u>all</u> causes.