CENTER FOR ADVANCED AVIATION SYSTEM DEVELOPMENT (CAASD)

# Tropospheric Corrections for GNSS Receivers

*The Atmosphere and its Effect on GNSS Systems*

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# Overview

- **Tropospheric Correction Model for SBAS Receivers**
  - **From WAAS MOPS, Reference [1]**
- **Tropospheric Correction Model for GBAS Receivers**
  - **From LAAS MOPS, Reference [2]**
- **Tropospheric Correction Model for GPS Receivers**
  - **Receiver manufacturers can use any tropospheric model**

# Tropospheric Correction Model for SBAS Receivers

# Tropospheric Delay Correction
## *(Section A.4.2.4 (Reference [1])*

- **The tropospheric delay correction, $TC_i$ (in meters), for satellite i takes the form**

$$TC_i = -\left(d_{hyd} + d_{wet}\right) \cdot m(El_i)$$

- **where**
  - **$d_{hyd}$ (m) = estimated zenith range delay caused by atmospheric gases in hydrostatic equilibrium**
  - **$d_{wet}$ (m) = estimated zenith range delay caused by water vapor respectively, and**
  - **$m(El_i)$ = mapping function to scale the delays to the actual satellite elevation angle ($El_i$)**



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# Calculation of $d_{hyd}$, and $d_{wet}$
*1 of 3*

- **Calculated from the receiver's height and estimates of five meteorological parameters:**
  - **Pressure [*P* (mbar)]**
  - **Temperature [*T* (K)]**
  - **Water vapor pressure [*e* (mbar)]**
  - **Temperature lapse rate [$\beta$ (K/m)]**
  - **Water vapor "lapse rate" ($\lambda$ (dimensionless))**

- **Values of each of the five meteorological parameters, applicable to the receiver latitude [$\phi$] and day-of-year [D] (starting 1 January of each year), are computed from the average and seasonal variation values given in Table A-2 of Reference [1] and upcoming slide**



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# Calculation of $d_{hyd}$, and $d_{wet}$

## *2 of 3*

- **Each parameter value $\xi$ is computed as:**

$$\xi(\phi, D) = \xi_0(\phi) - \Delta\xi(\phi) \cdot \cos\left(\frac{2\pi(D - D_{\min})}{365.25}\right)$$

- **where $D_{min}$ = 28 for northern latitudes, $D_{min}$ = 211 for southern latitudes, and $\xi_0$, $\Delta\xi$ are the average and seasonal variation values for the particular parameter at the receiver's latitude**
- **For latitudes $|\phi| \leq 15°$ and $|\phi| \geq 75°$, values for $\xi_0$ and $\Delta\xi$ are taken directly from Table A-2 of Reference [1]**
- **For latitudes in the range $15° < |\phi| < 75°$, values for $\xi_0$ and $\Delta\xi$ at the receiver's latitude are each pre-calculated by linear interpolation between values for the two closest latitudes [$\phi_i$, $\phi_{i+1}$] in Table A-2 of Reference [1]:**

$$\xi_0(\phi) = \xi_0(\phi_i) + \left[\xi_0(\phi_{i+1}) - \xi_0(\phi_i)\right] \cdot \frac{(\phi - \phi_i)}{(\phi_{i+1} - \phi_i)}$$

$$\Delta\xi(\phi) = \Delta\xi(\phi_i) + \left[\Delta\xi(\phi_{i+1}) - \Delta\xi(\phi_i)\right] \cdot \frac{(\phi - \phi_i)}{(\phi_{i+1} - \phi_i)}$$



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# Calculation of $d_{hyd}$, and $d_{wet}$

### *3 of 3*

- **Zero-altitude zenith delay terms [$z_{hyd}$, $z_{wet}$ (m)] are calculated as:**

$$z_{hyd} = \frac{10^{-6} k_1 R_d P}{g_m} \qquad z_{wet} = \frac{10^{-6} k_2 R_d}{g_m(\lambda + 1) - \beta R_d} \cdot \frac{e}{T}$$

- **Where $k_1$ = 77.604 K/mbar, $k_2$ = 382000 $K^2$/mbar, $R_d$ = 287.054 J/(kg·K), and $g_m$ = 9.784 m/$s^2$.**
- **[$d_{hyd}$, $d_{wet}$] are calculated as:**

$$d_{hyd} = \left(1 - \frac{\beta H}{T}\right)^{\frac{g}{R_d \beta}} \cdot z_{hyd} \qquad d_{wet} = \left(1 - \frac{\beta H}{T}\right)^{\frac{(\lambda+1)g}{R_d \beta} - 1} \cdot z_{wet}$$

- **Where $g$ = 9.80665 m/$s^2$ and the receiver's height, [$H$] is expressed in units of meters above mean sea level.**



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# Meteorological Parameters for Tropospheric Delay (Table A-2 of Reference [1])

| | Average | | | | |
|---|---|---|---|---|---|
| **Latitude (°)** | $P_0$ *(mbar)* | $T_0$ *(K)* | $e_0$ *(mbar)* | $\beta_0$ *(K/m)* | $\lambda_0$ |
| 15° or less | 1013.25 | 299.65 | 26.31 | 6.30e-3 | 2.77 |
| 30 | 1017.25 | 294.15 | 21.79 | 6.05e-3 | 3.15 |
| 45 | 1015.75 | 283.15 | 11.66 | 5.58e-3 | 2.57 |
| 60 | 1011.75 | 272.15 | 6.78 | 5.39e-3 | 1.81 |
| 75° or greater | 1013.00 | 263.65 | 4.11 | 4.53e-3 | 1.55 |
| | **Seasonal Variation** | | | | |
| **Latitude (°)** | $\Delta P$ *(mbar)* | $\Delta T$ *(K)* | $\Delta e$ *(mbar)* | $\Delta\beta$ *(K/m)* | $\Delta\lambda$ |
| 15° or less | 0.00 | 0.00 | 0.00 | 0.00e-3 | 0.00 |
| 30 | -3.75 | 7.00 | 8.85 | 0.25e-3 | 0.33 |
| 45 | -2.25 | 11.00 | 7.24 | 0.32e-3 | 0.46 |
| 60 | -1.75 | 15.00 | 5.36 | 0.81e-3 | 0.74 |
| 75° or greater | -0.50 | 14.50 | 3.39 | 0.62e-3 | 0.30 |

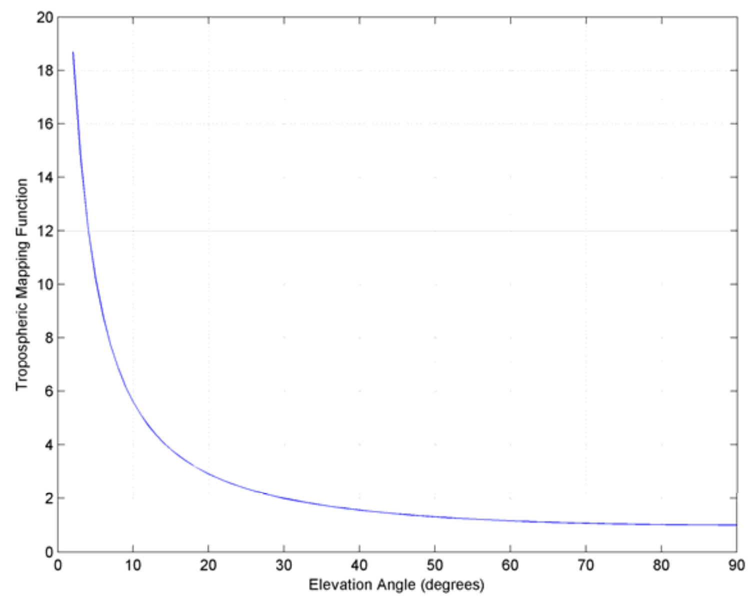
# Tropospheric Mapping Function m(El)

- **For $El_i \geq 4$ degrees:**

$$m(El_i) = \frac{1.001}{\sqrt{0.002001 + \sin^2(El_i)}}$$

- **For $El_i \geq 2$ degrees [Reference 3]**

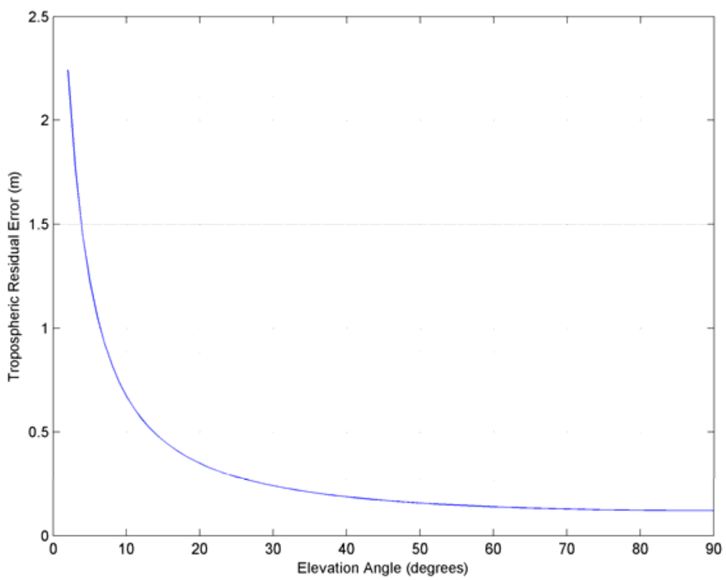
$$m(El_i) = \left( \frac{1.001}{\sqrt{0.002001 + \sin^2(El_i)}} \right) \cdot \left( 1 + 0.015 \cdot \left( MAX \begin{bmatrix} 0 \\ (4^\circ - El_i) \end{bmatrix} \right)^2 \right)$$



# Tropospheric Residual Error

- **Tropospheric residual Error in meters**

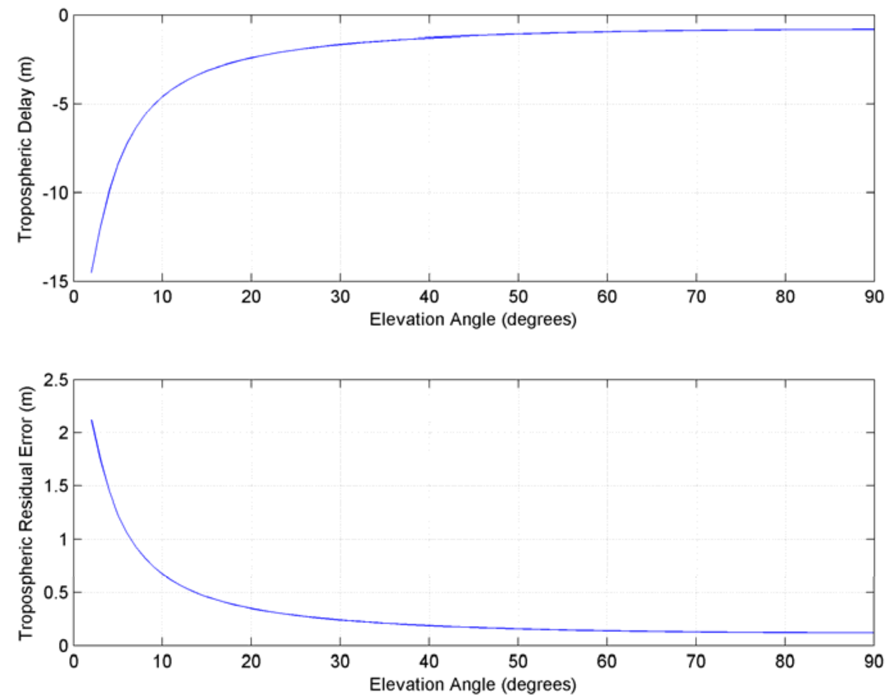
$$\sigma_{i,tropo} = \left(\sigma_{TVE} \cdot m(El_i)\right)$$




- **Where**
  - **$\sigma_{TVE}$ = 0.12 m (zenith tropospheric residual error)**
  - **$m(El_i)$ = mapping function**

# Example
# Santiago, Chile

- **Lat/Lon: 33:28S/70:45W**
- **Date: April 15, 2008**
- **Day of Year = 106**
- **Aircraft Altitude = 30,000 ft (9,144 meters)**



# Tropospheric Correction Model for LAAS/GBAS Receivers

# Tropospheric Correction in meters

- **Paragraph: 3.3.2.14 [Reference 2]**

$$TC(El) = TC_V . m(El), \text{ where}$$

$$TC_V = 10^{-6} N_R h_0 \left( 1 - e^{-\frac{\Delta h}{h0}} \right) = \text{Zenith Delay (meters)}$$

$$m(El) = \frac{1}{\sqrt{0.002 + \sin^2(El)}} = \text{Mapping Function}$$

$El$ = Elevation Angle (radians)

$N_R$ = Refractivity index transmitted by ground system

$\Delta h$ = Difference in altitude between airborne and ground subsystems (meters)

$h_0$ = Tropospheric scale height transmitted by the ground subsystem (meters)

# Residual Tropospheric Error (meters)

- **Paragraph: 3.3.2.15 [Reference 2]**

$$\sigma_{tropo}(El) = \sigma_{tropo,V} \cdot m(El), \text{ where}$$

$$\sigma_{tropo,V} = 10^{-6}\sigma_N h_0 \left(1 - e^{-\frac{\Delta h}{h0}}\right) = \text{Error in Zenith Delay (meters)}$$

$$m(El) = \frac{1}{\sqrt{0.002 + \sin^2(El)}} = \text{Mapping Function}$$

$El$ = Elevation Angle (radians)

$\sigma_N$ = Refractivity index error transmitted by ground system

$\Delta h$ = Difference in altitude between airborne and ground subsystems (meters)

$h_0$ = Tropospheric scale height transmitted by the ground subsystem (meters)

# References

1. RTCA, Inc., *Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System*, DO-229D, RTCA, Inc., Washington, D.C., 2006.

2. RTCA, Inc., *Minimum Operational Performance Standards for Local Area Augmentation System*, DO-245A, RTCA, Inc., Washington, D.C., 2004.

3. Bellingham, S., "A Modified Tropospheric Model for Satellite Elevation Angles to 2º," Presentation to RTCA SC-159 WG-2, March 9, 2005.